

Exercise 15

For the following exercises, determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry.

$$f(x) = 2x^2 - 10x + 4$$

Solution

Begin by factoring the coefficient of x^2 .

$$f(x) = 2(x^2 - 5x + 2)$$

In order to write this quadratic function in vertex form, it's necessary to complete the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -5$, which means $B = -\frac{5}{2}$ and $B^2 = \frac{25}{4}$. Add and subtract $\frac{25}{4}$ on the right side within the parentheses and use the identity so that x appears in only one place.

$$\begin{aligned} f(x) &= 2 \left[\left(x^2 - 5x + \frac{25}{4} \right) + 2 - \frac{25}{4} \right] \\ &= 2 \left[\left(x + \left(-\frac{5}{2} \right) \right)^2 - \frac{17}{4} \right] \\ &= 2 \left(x - \frac{5}{2} \right)^2 - \frac{17}{2} \end{aligned}$$

Therefore, the vertex of the parabola is $\left(\frac{5}{2}, -\frac{17}{2}\right)$. The axis of symmetry is $x = \frac{5}{2}$, and the minimum (because the coefficient of x^2 is positive) is $y = -\frac{17}{2}$.

